Properties of many-body localized phase

Maksym Serbyn

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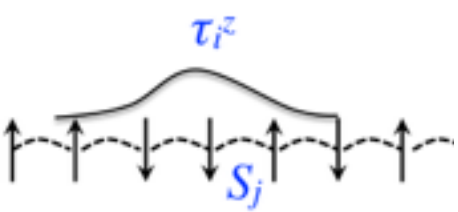

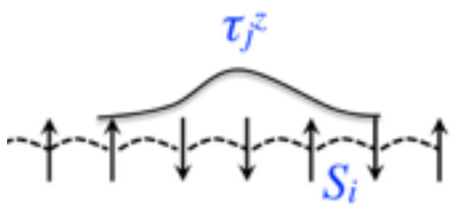

QMATH13

Atlanta, 2016

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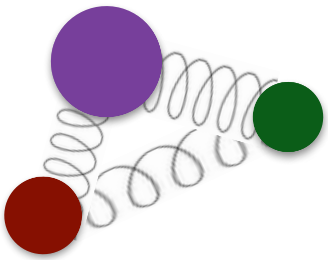
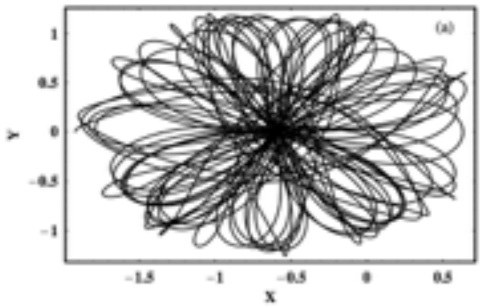
Motivation: MBL as a new universality class

- **Classical systems:**

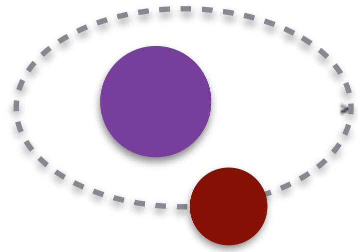
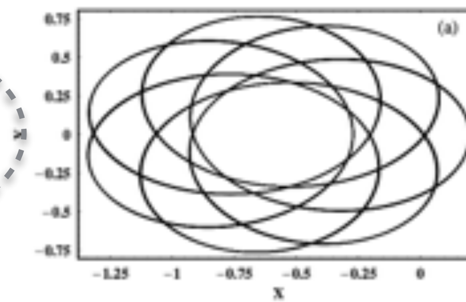
<u>Ergodic</u>

ergodicity from chaos

<u>Integrable</u>

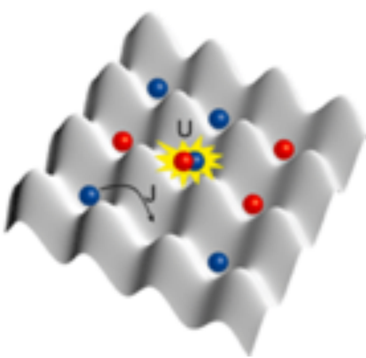
stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]

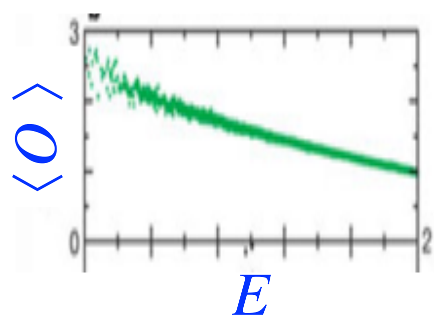



- **Quantum systems:**

<u>Thermalizing</u>

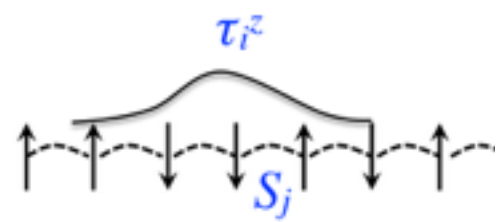
ETH mechanism





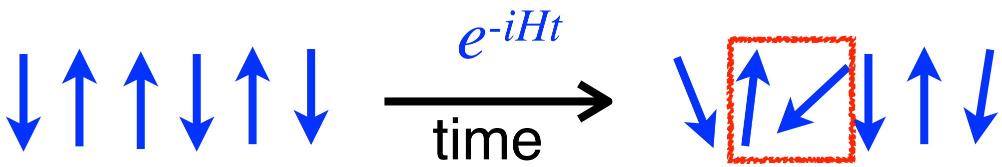
<u>Many-body localized</u>

emergent integrability
stable to weak perturbations



Thermalizing systems: ETH

- Thermalization:

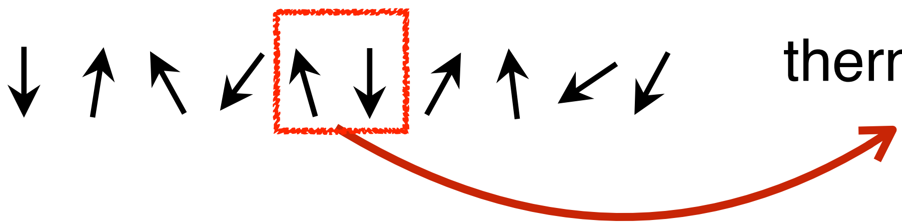


subsystem in thermal state

- Mechanism:

  Eigenstate Thermalization Hypothesis: [Deutsch'91] [Srednicki'94] [Rigol,Dunjko,Olshanii'08]

  property of eigenstates

$|\lambda\rangle =$ 

thermal density matrix

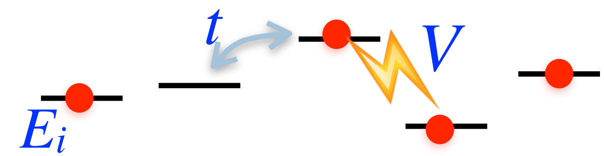
$\rho_\lambda = e^{-H/T}$

- Works in many cases

  Many open questions: timescale, other mechanisms?..

Many-body localized phase

- MBL = localized phase with interactions



- Perturbative arguments for existence of MBL phase:

[Basko,Aleiner,Altshuler'05][Gornyi,Polyakov,Mirlin'05]

- Numerical evidence for MBL: [Oganesyan,Huse'08] [Pal,Huse'10] [Znidaric,Prosen'08] [Monthus, Garel'10][Bardarson,Pollman,Moore'12] [MS, Papic, Abanin'13,'14] [Kjall et al'14]

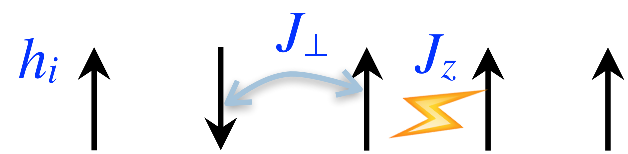
Non-thermalizing MBL phase exists!

Properties of MBL phase?
Why thermalization breaks down?

Universal Hamiltonian of MBL phase

- If model is in MBL phase, rotate basis

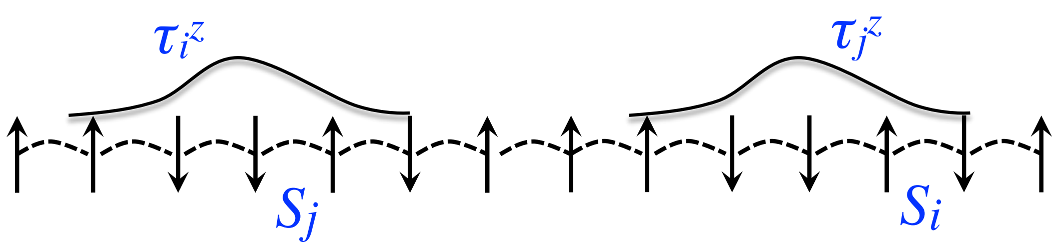
$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + h_i S_i^z$$



- New spins: $\tau_i = U^\dagger \, S_i \, U$ are quasi-local; form complete set

$$\hat{H} = \sum_i H_i \tau_i^z + \sum_{ij} H_{ij} \tau_i^z \tau_j^z + \sum_{ijk} H_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

$H_{ij} \propto \exp(-|i-j|/\xi)$



- Consequences: no transport, ETH breakdown, universal dynamics

[MS, Papic, Abanin, PRL'13]
[Huse, Oganesyan, PRB'14]
[Imbrie, arXiv:1403.7837]

Properties of MBL phase

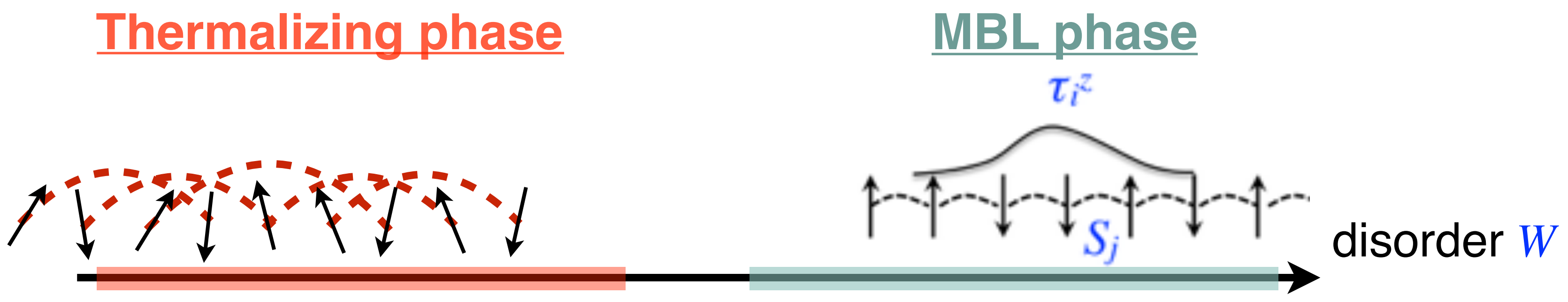
- Transport:

Diffusion
Entanglement light cone

??

- Matrix elements:

- Eigenstate properties:



Dynamics in MBL phase

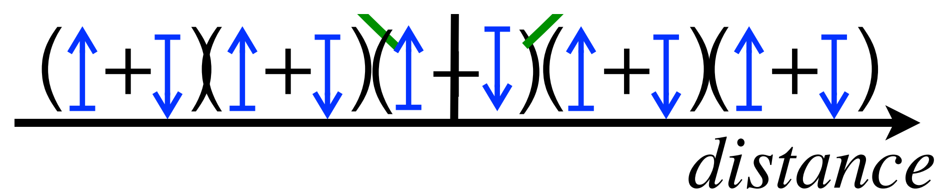
$$\hat{H} = \sum_i H_i \tau_i^z + \sum_{ij} H_{ij} \tau_i^z \tau_j^z + \sum_{ijk} H_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots \qquad H_{ij} \propto J e^{-|i-j|/\xi}$$

- Dephasing dynamics

- Phases randomize

  on distance $x(t)$:

$$tH_{ij} = tJ\,\text{exp}(-x/\xi) \sim 1$$

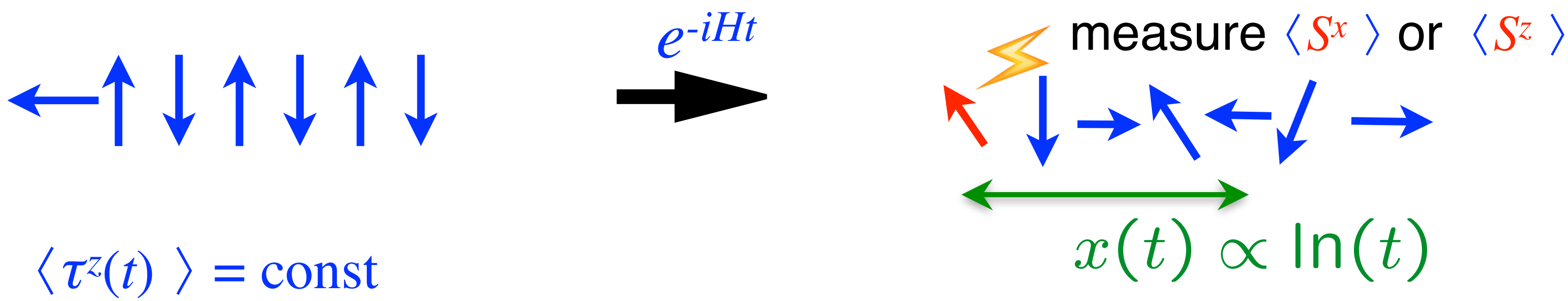


- Explains logarithmic growth of entanglement

[MS, Papic, Abanin, PRL'13]

- Dynamics of local observables?

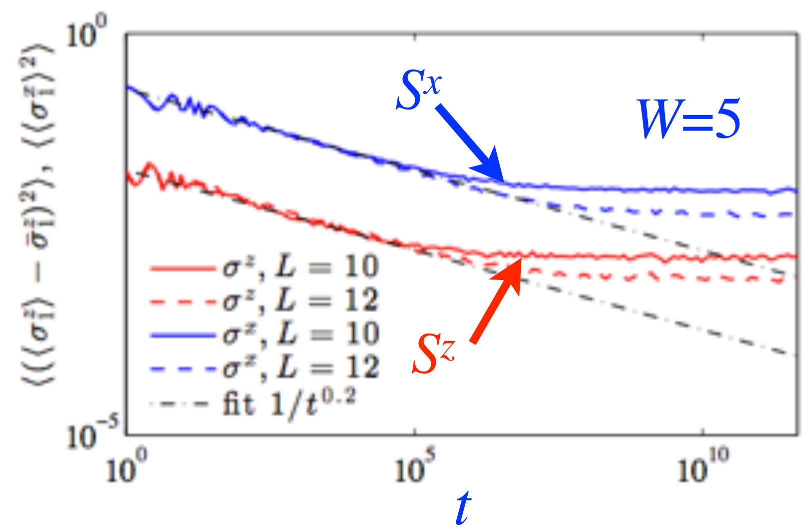
Local observables in a quench



- $\langle \tau^z(t) \rangle = \text{const}$
- $\langle \tau^x(t) \rangle = \rho_{\uparrow\downarrow}(t)$ = [sum of $N(t) = 2^{x(t)}$ oscillating terms]
- Decay of oscillations of $\langle \tau^x(t) \rangle$: $|\langle \tau_k^x(t) \rangle| \propto \frac{1}{\sqrt{N(t)}} = \frac{1}{(tJ)^a}$

$$\left| \langle \hat{\mathcal{O}}(t) \rangle - \langle \mathcal{O}(\infty) \rangle \right| \sim \frac{1}{t^a}$$

memory of initial state



[MS, Papic, Abanin, PRB'14]

Properties of MBL phase

- Transport:

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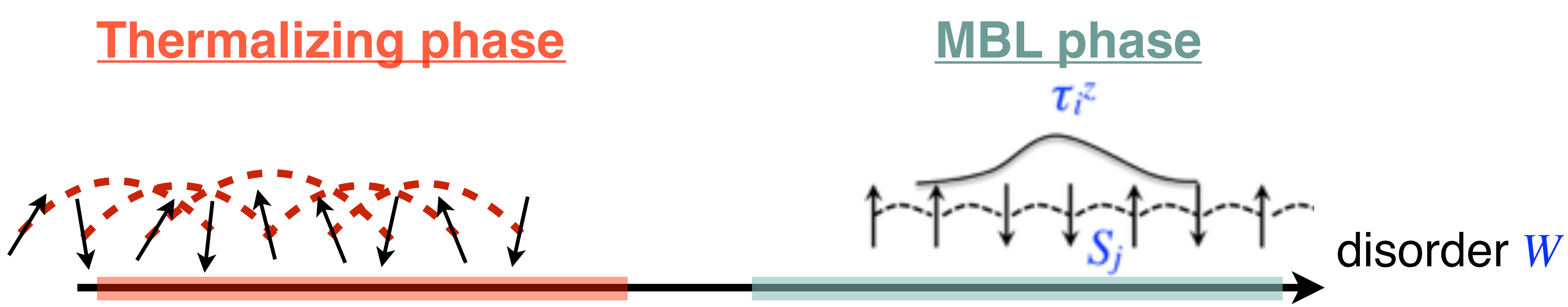
No transport
Log-growth of entanglement

- Matrix elements

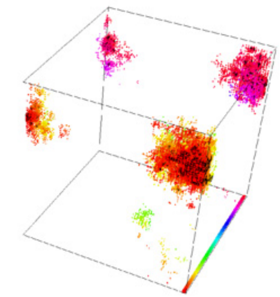
ETH ansatz, typicality

??



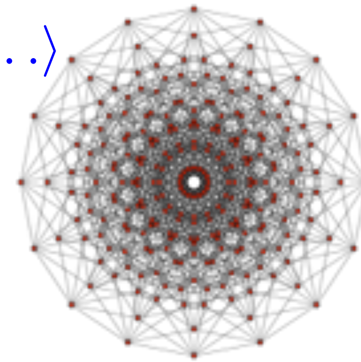
Structure of many-body wave function

- Single-particle localization: $\psi(x)$



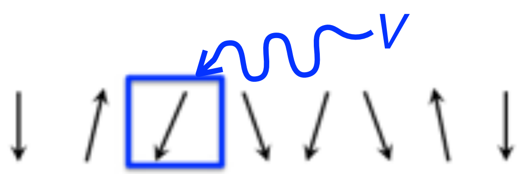
- Many-body wave function:

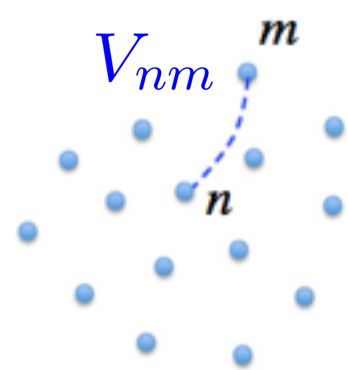
$$|\psi\rangle = \sum_{\sigma=\uparrow,\downarrow} \psi_{\sigma_1\sigma_2\ldots} |\sigma_1\sigma_2\ldots\rangle$$



Problems: basis-dependent,
not related to observables

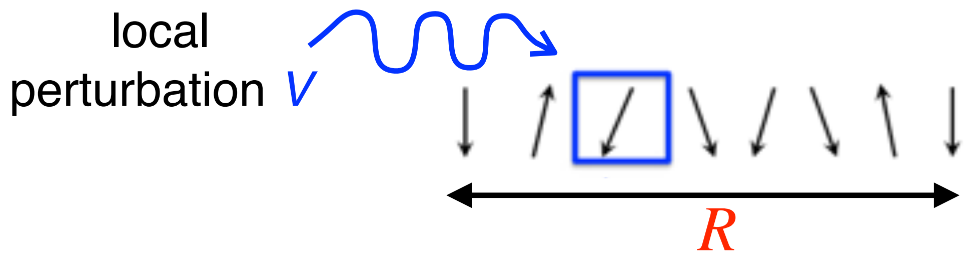
- Alternative: “wave function” created by *V*





$$H|n\rangle = E_n|n\rangle \qquad \psi_n(m) = \langle m|V|n\rangle$$

Matrix elements of local operators



<u>ETH ansatz</u>

$$\langle i|S^z|j\rangle = e^{-S(E,R)/2} f(E_i, E_j) R_{ij}$$

[Srednicki'99]

narrow distribution:

$$\langle i|S^z|j\rangle \sim 1/\sqrt{2^R}$$

<u>Local integrals of motion</u>

$$S^z = \sum_{\{\alpha\}} \hat{\tau}^{\{\alpha\}} \hat{B}^{\{\alpha\}}[\tau^z]$$

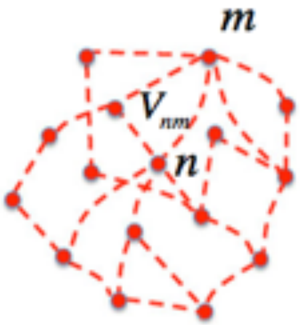
$$\langle i|S^z|j\rangle$$

broad distribution:

$$\langle i|S^z|j\rangle \sim \exp(-\kappa' R)$$

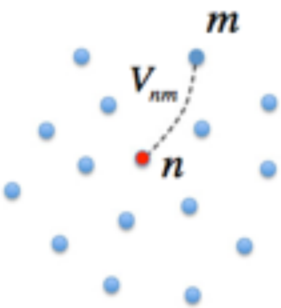
Fractal analysis of matrix elements

- Fractal dimensions from scaling of $P_q = \sum_m \langle |V_{nm}|^{2q} \rangle \propto \frac{1}{\mathcal{D}^{\tau_q}}$



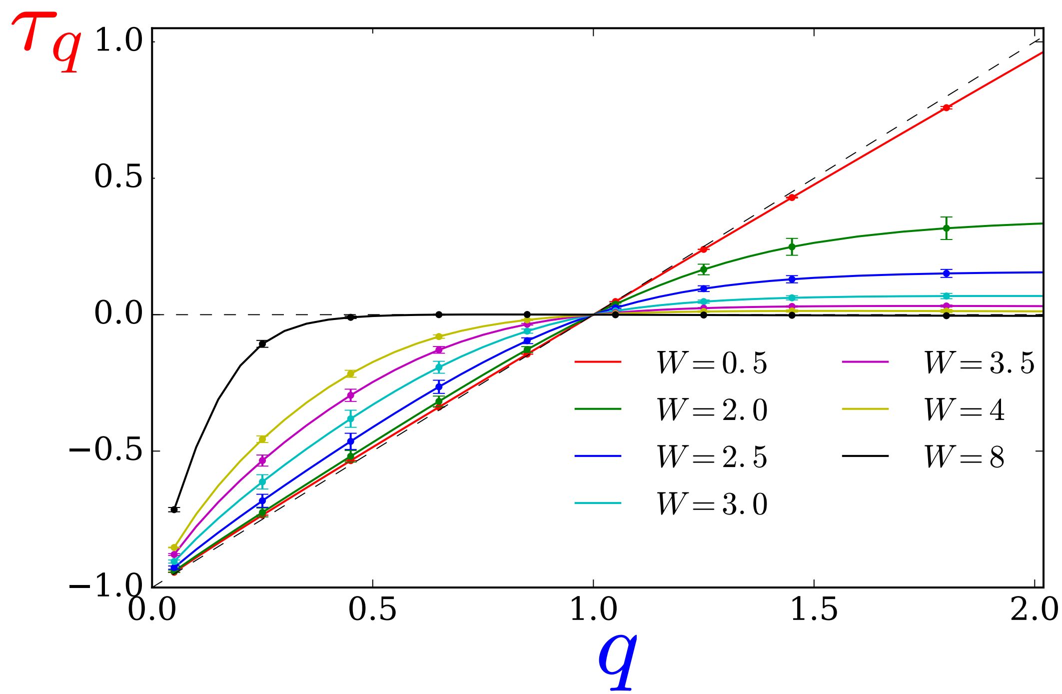
Ergodic phase

$\tau_q = q - 1$



MBL phase

$\tau_{q>q_c} = 0$



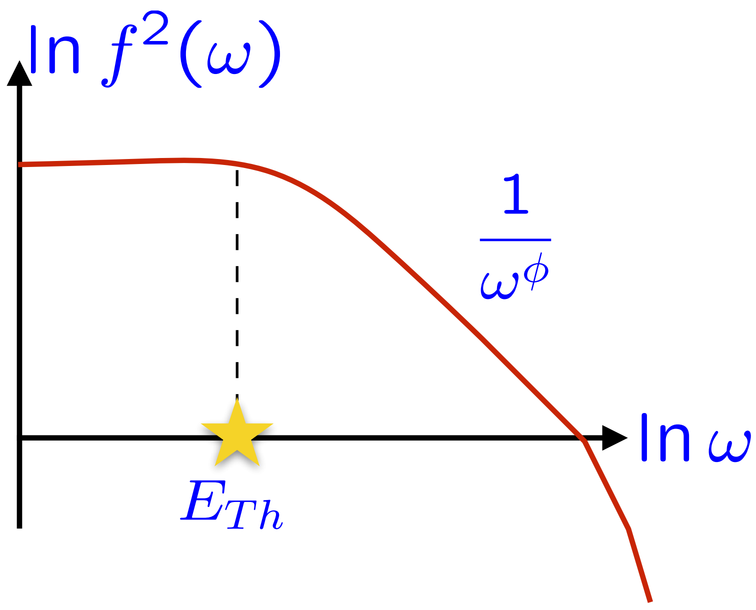
- “Frozen” fractal spectrum in MBL: $\langle \ln V_{nm} \rangle \propto -\kappa L$

Energy structure of matrix elements

- Spectral function $f^2(\omega) = e^{S(E)} \langle |V_{nm}|^2 \delta(\omega - (E_m - E_n)) \rangle$

- Related to dynamics: $\langle \alpha | V(t) V(0) | \alpha \rangle_c \approx \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} f^2(\omega)$

- Thermalizing phase:



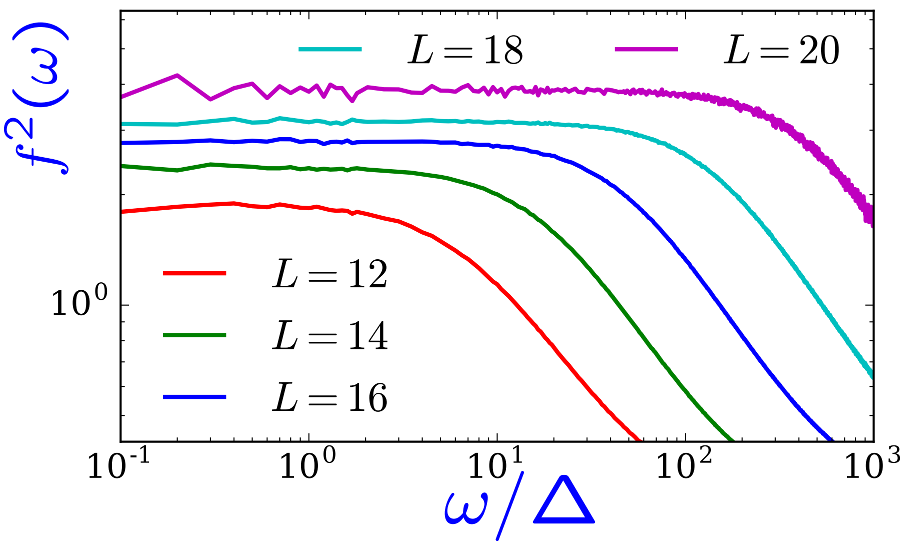
$$\langle \alpha | V(t) V(0) | \alpha \rangle_c \propto \frac{1}{t^{1-\phi}}$$

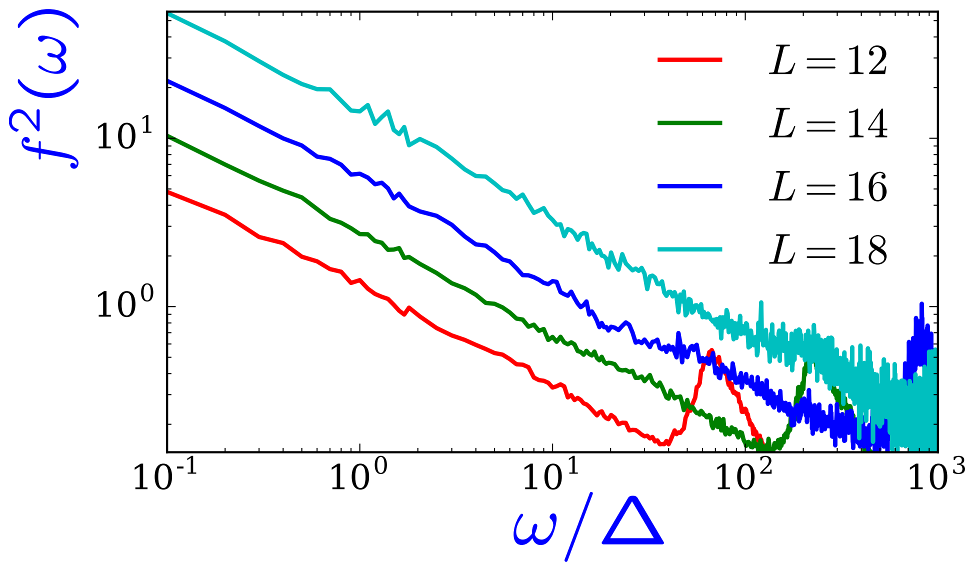
$$E_{Th} \propto \frac{1}{L^{1/(1-\phi)}}$$

more details:

[arXiv:1610.02389]

Numerical results for spectral function:





- MBL phase: Thouless energy < level spacing
- Breakdown of typicality: $\log\langle V_{nm}\rangle \neq \langle \log V_{nm}\rangle$

Properties of MBL phase

- Transport:

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Log-growth of entanglement

- Matrix elements:

ETH ansatz, typicality

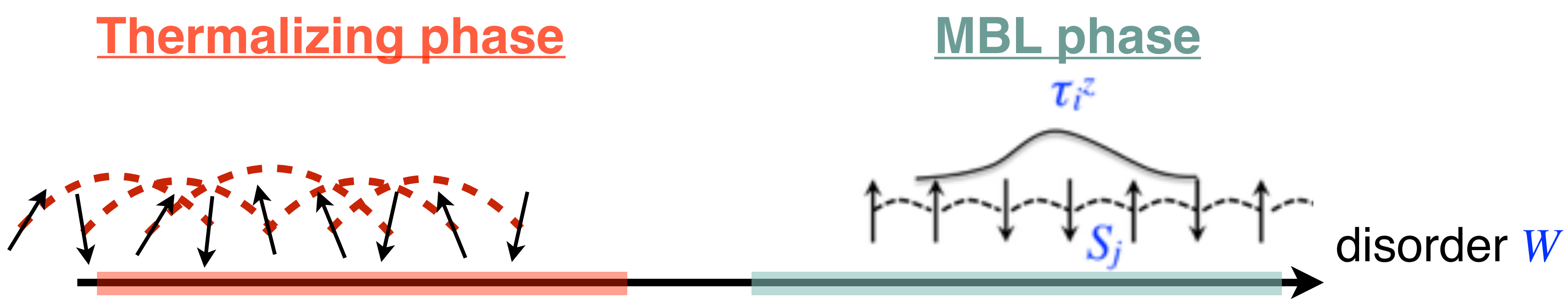
broad distribution
strong fractality

- Eigenstate properties:

volume-law entanglement

“flat” entanglement spectrum

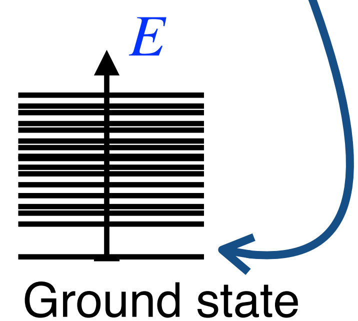
??



Beyond entanglement

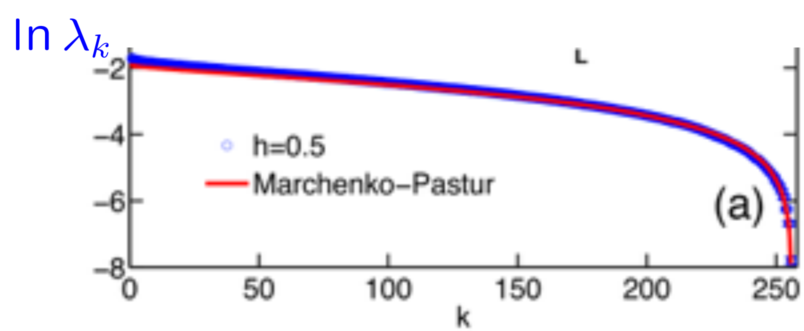
- Gapped ground states: area-law $S_{\mathrm{ent}}(L) \sim const$ in 1d
- Excited eigenstates: volume-law

$S_{\mathrm{ent}}(L) \sim L$ in 1d



- MBL: area-law entanglement
  Q: Difference with gapped ground states?
- Entanglement spectrum $\{\lambda_i\}$ $S_{\mathrm{ent}} = -\sum_i \lambda_i \log \lambda_i$
- "Flat" in ergodic states:

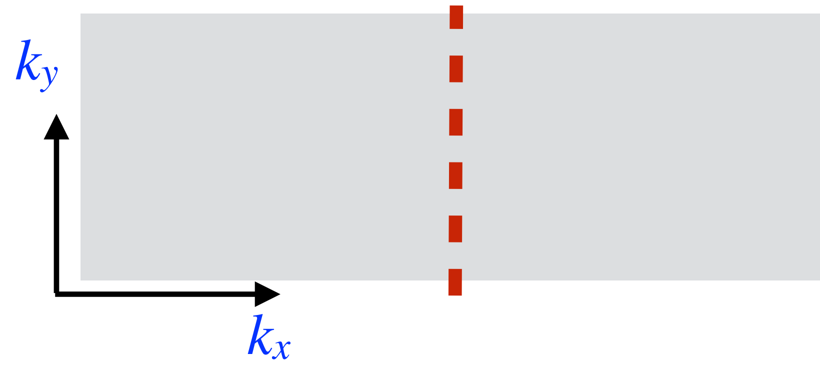
[Marchenko&Pastur'67]
[Yang,Chamon,Hamma&Muciolo'15]



Entanglement spectrum: probes boundary

- Quantum Hall wave function:

  k_y to organize ES
  [Li & Haldane]



- MBL phase: conserved quantities label ES

$$|\uparrow\uparrow\uparrow\uparrow\rangle = c_0\,|\uparrow\uparrow\rangle|\uparrow\uparrow\rangle + e^{-\kappa}\,\underbrace{|\uparrow\downarrow\rangle|\uparrow\uparrow\rangle}_{r=1} + e^{-2\kappa}\,\underbrace{|\uparrow\downarrow\rangle|\downarrow\uparrow\rangle}_{r=2} + \ldots$$

$$+\; e^{-4\kappa}\,\underbrace{|\downarrow\downarrow\rangle|\downarrow\downarrow\rangle}_{r=4} + \;.....$$

- Coefficients decay as $\quad |C_{\uparrow\ldots\uparrow\underbrace{\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow}_{r}\uparrow\ldots\uparrow}| \propto e^{-\kappa r}$

Power-law entanglement spectrum

- Hierarchical structure of $\rho_L = \sum_{r=0}^{L} |\psi^{(r)}\rangle\langle\psi^{(r)}|$

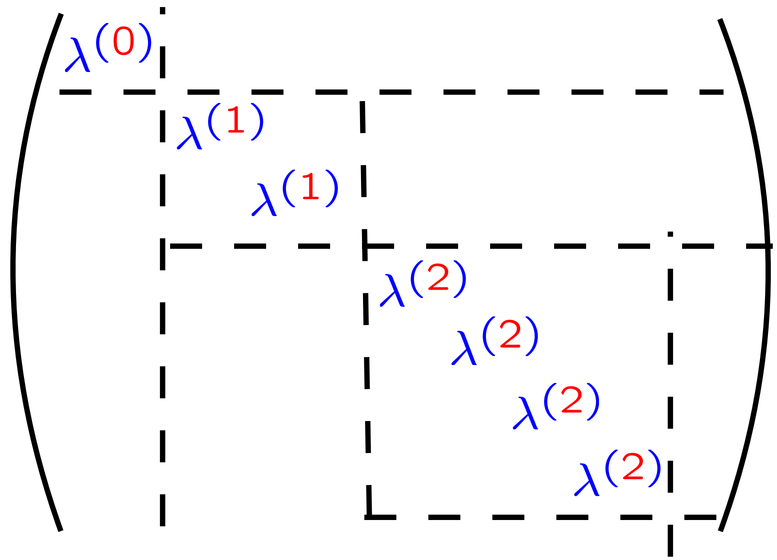
$$\langle\psi^{(r)}|\psi^{(r)}\rangle \propto e^{-2\kappa r}$$

but non-orthogonal

- Orthogonalize perturbatively

$$\lambda^{(r)} \propto e^{-4\kappa r}$$

multiplicity is 2^r

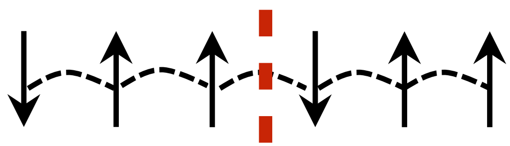


- Power-law entanglement spectrum

$$\lambda_k \propto \frac{1}{k^\gamma} \qquad \gamma \approx \frac{4\kappa}{\ln 2}$$

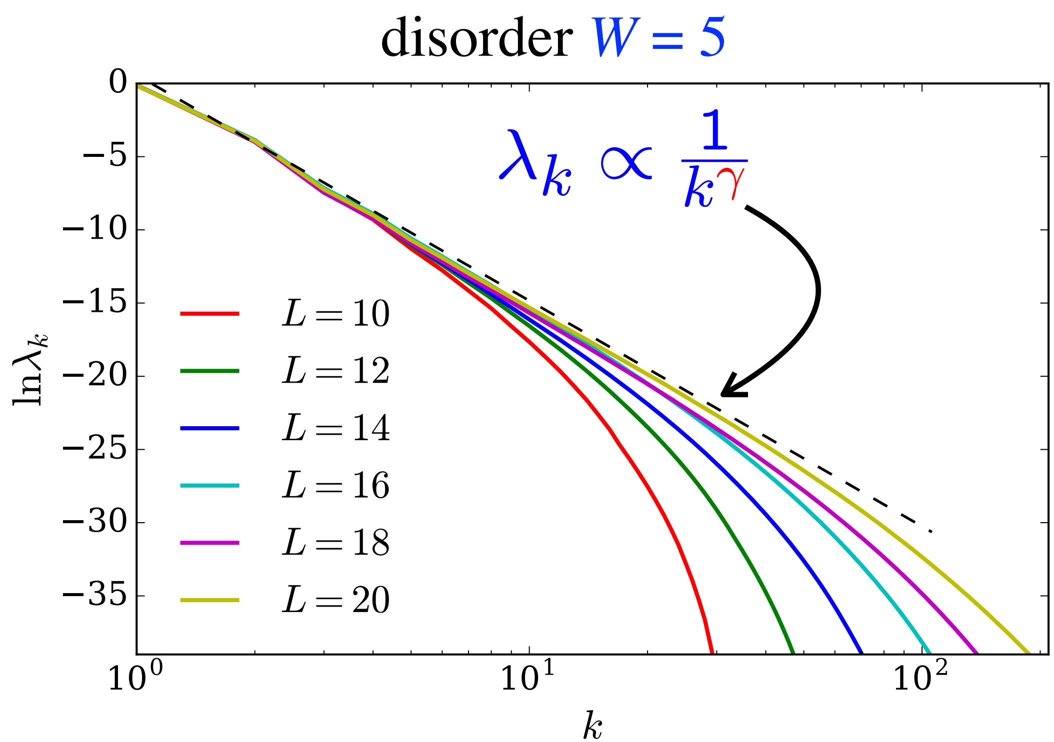
Numerics for XXZ spin chain

- Numerical studies for XXZ spin chain, $J_\perp = J_z = 1$



$$H = \sum_i (h_i S_i^z + J_\perp S_i^+ S_{i+1}^- + h.c.) + \sum_i J_z S_i^z S_{i+1}^z$$

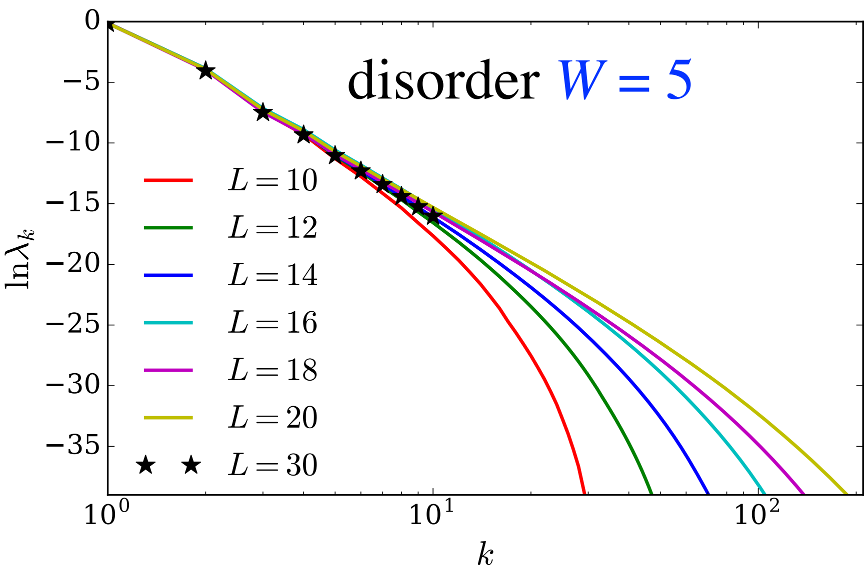
- Power law entanglement spectrum:

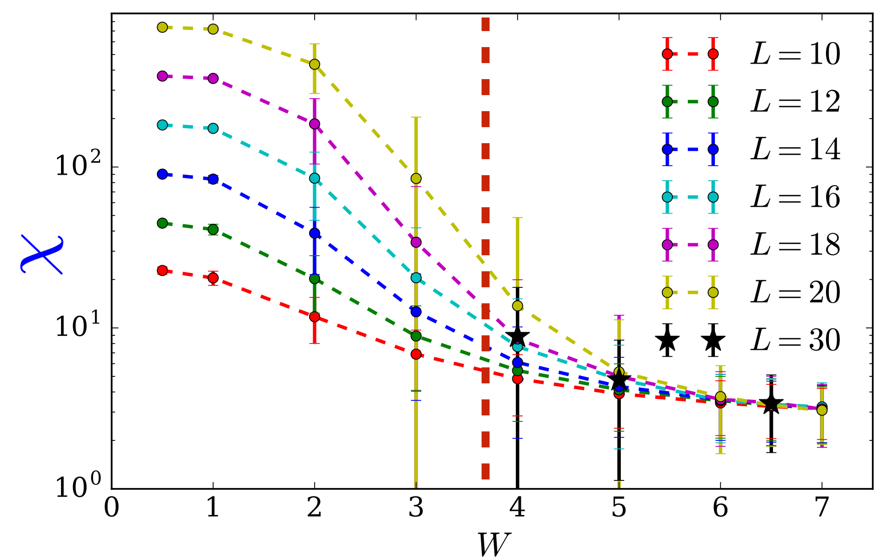


more details in:
[arXiv:1605.05737]

Estimates for the bond dimension

- Large γ → MPS error $\propto 1/\chi^{\gamma-1}$ can be small
- Implementation of DMRG for highly excited states:





more details:
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also: [Yu et al arXiv:1509.01244] [Lim&Sheng arXiv:1510.08145]
[Pollmann et al arXiv:1509.00483] [Kennes&Karrasch arXiv:1511.02205]

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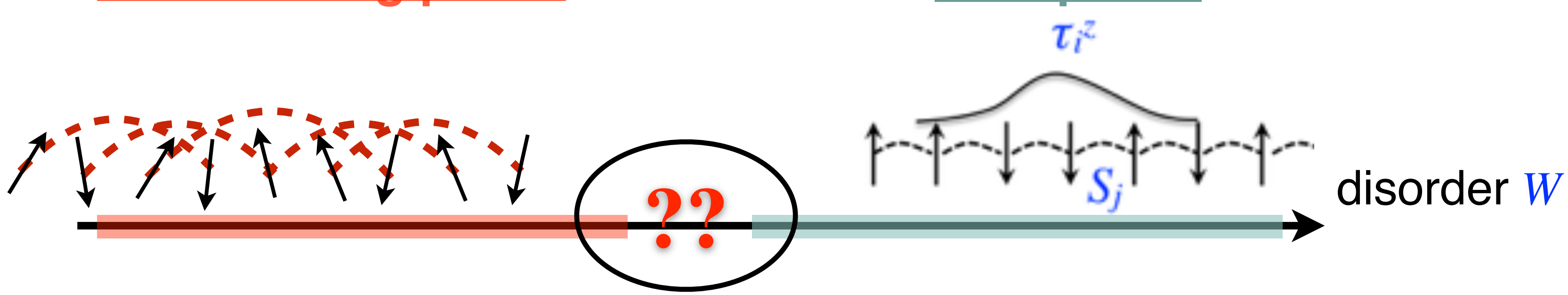
"flat" entanglement spectrum

area-law entanglement

power-law entanglement spectrum

<u>**Thermalizing phase**</u> <u>**MBL phase**</u>



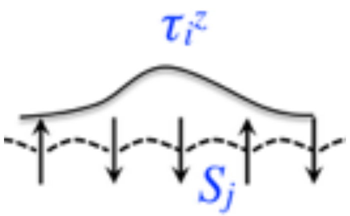
Summary and outlook

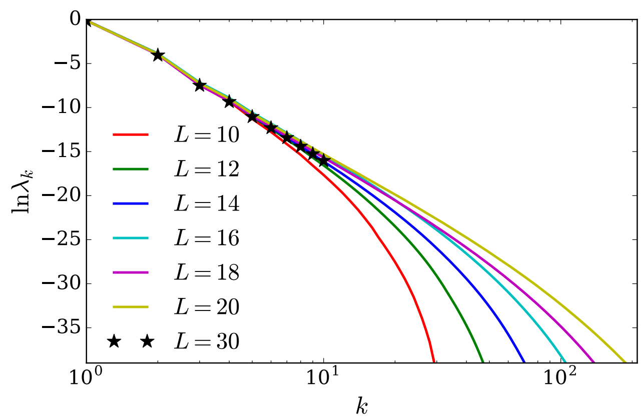
- MBL: new universality class of non-thermalizing systems

- Properties: dynamics, matrix elements, entanglement

$$\left|\langle\hat{\mathcal{O}}(t)\rangle - \langle\mathcal{O}(\infty)\rangle\right| \sim \frac{1}{t^a}$$

$$\langle i|S^z|j\rangle \sim \exp(-\kappa' R)$$





PRL 110, 260601 (2013)
PRL 111, 127201 (2013)
PRL 113, 147204 (2014)
PRB 90, 174302 (2014)
PRX 5, 041047 (2015)
PRB 93, 041424 (2016)
arXiv:1605.05737
arXiv:1610.02389

- Questions: MBL in d>1, symmetries, MPS/MPO description breakdown of MBL, mobility edge

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UC Berkeley

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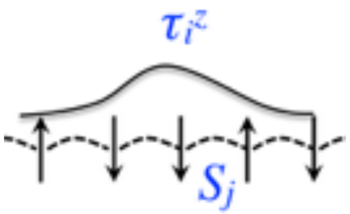

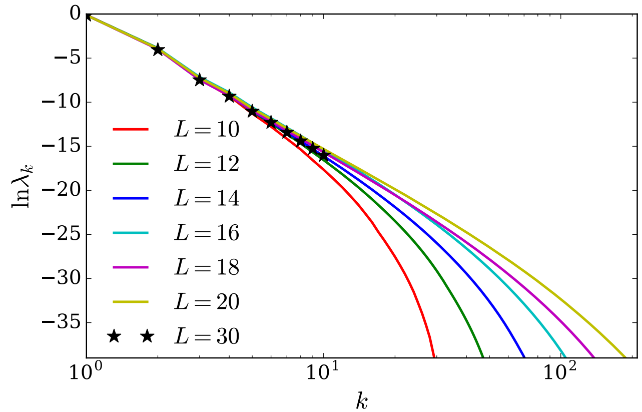

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